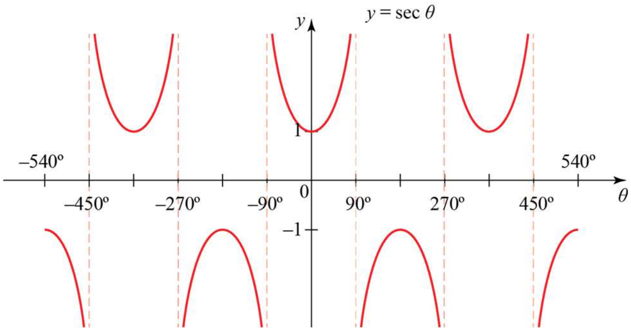
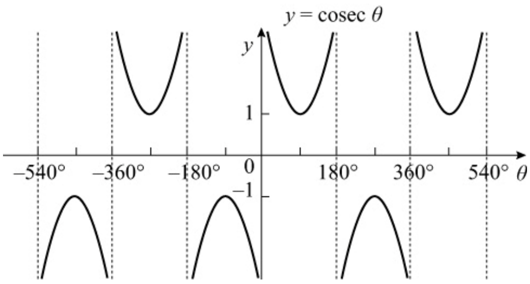
Exercise 3B

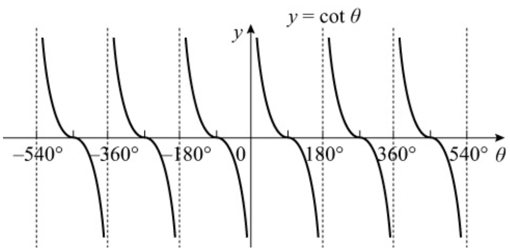
1 a



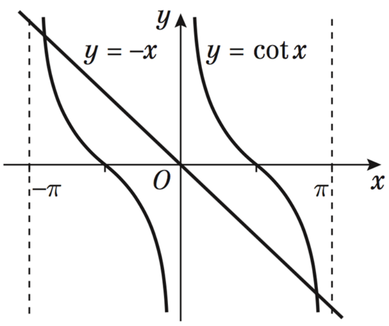
b



c

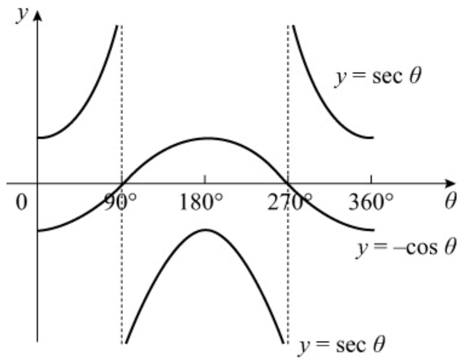


2 a



b 2 solutions

3 a



b You can see that the graphs of $y = \sec\theta$ and $y = -\cos\theta$ do not meet, so $\sec\theta = -\cos\theta$ has no solutions.

The same result can be found algebraically

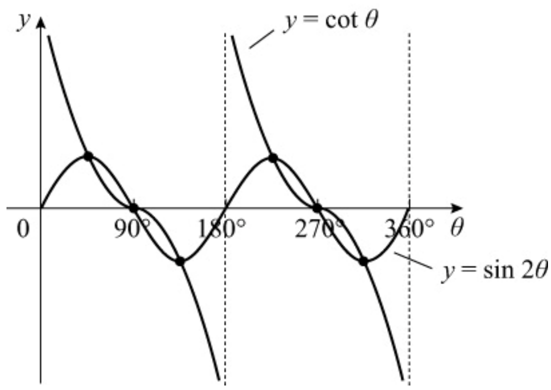
$\sec\theta = -\cos\theta$

$\Rightarrow \dfrac{1}{\cos\theta} = -\cos\theta$

$\Rightarrow \cos^2\theta = -1$

There are no solutions of this equation for real θ.

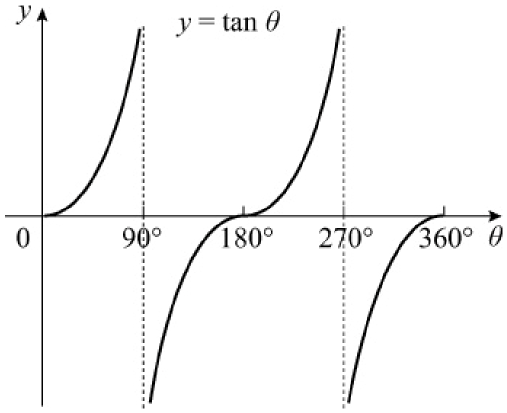
4 a

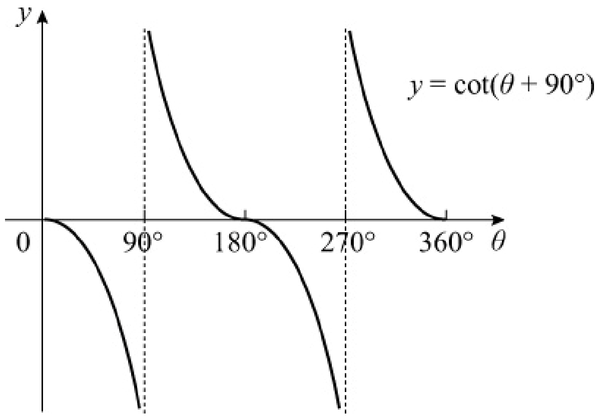


b The curves meet at the maxima and minima of $y = \sin 2\theta$, and on the θ-axis at odd integer multiples of 90°.

In the interval $0 \le \theta \le 360°$ there are 6 intersections. So there are 6 solutions of $\cot\theta = \sin 2\theta$ in the interval $0 \le \theta \le 360°$

5 a





b $y = \cot(\theta + 90°)$ is a reflection in the θ-axis of $y = \tan\theta$, so
$\cot(\theta + 90°) = -\tan\theta$

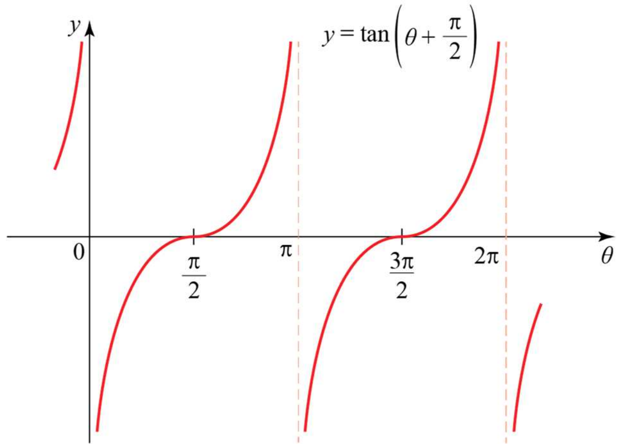
6 a i The graph of $y = \tan\left(\theta + \frac{\pi}{2}\right)$ is the same as that of $y = \tan\theta$ translated by the vector $\begin{pmatrix} -\frac{\pi}{2} \\ 0 \end{pmatrix}$, i.e by $\frac{\pi}{2}$ to the left.

ii The graph of $y = \cot(-\theta)$ is the same as that of $y = \cot\theta$ reflected in the y-axis.

iii The graph of $y = \operatorname{cosec}\left(\theta + \frac{\pi}{4}\right)$ is the same as that of $y = \operatorname{cosec}\theta$ translated by the vector $\begin{pmatrix} -\frac{\pi}{4} \\ 0 \end{pmatrix}$

iv The graph of $\sec\left(\theta - \frac{\pi}{4}\right)$ is the same as that of $y = \sec\theta$ translated by the vector $\begin{pmatrix} \frac{\pi}{4} \\ 0 \end{pmatrix}$

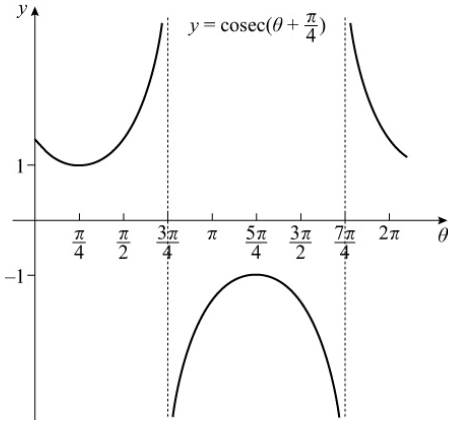
b

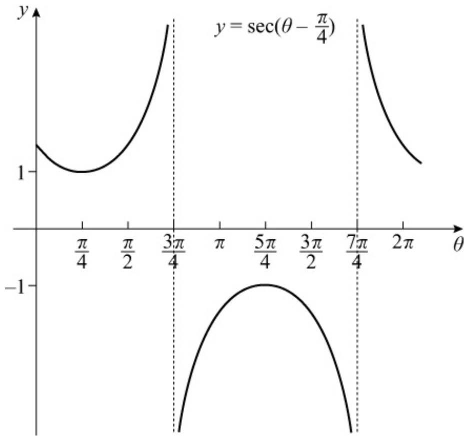


(reflection of $y = \cot\theta$ in the y-axis)

$\tan\left(\theta + \frac{\pi}{2}\right) = \cot(-\theta)$

6 b





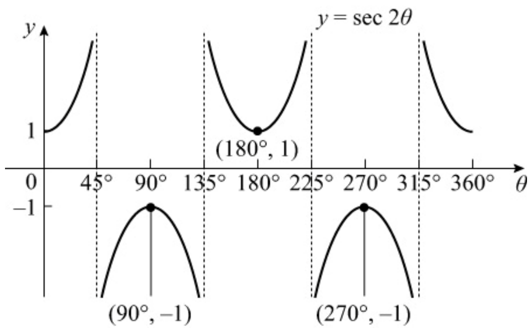
$$\mathrm{cosec}\left(\theta+\frac{\pi}{4}\right)=\sec\left(\theta-\frac{\pi}{4}\right)$$

7 a A stretch of $y=\sec\theta$ in the θ direction with scale factor $\frac{1}{2}$

Minimum at (180°,1)

Maxima at (90°,–1) and (270°,–1)

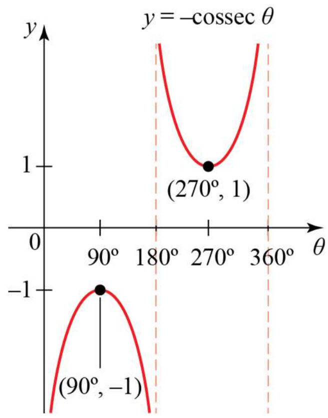
It meets the y-axis at (0, 1)



b Reflection in θ-axis of $y=\mathrm{cosec}\,\theta$

Minimum at (270°, 1)

Maximum at (90°, -1)

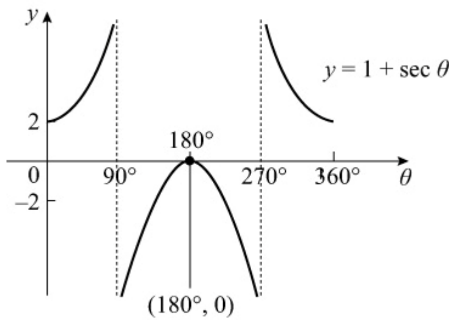


c Translation of $y=\sec\theta$ by the vector $\begin{pmatrix}0\\1\end{pmatrix}$, i.e. +1 in the y direction.

It meets x-axis at (180°, 0)

There is a maximum at (180°, 0)

It meets the y-axis at (0, 2)

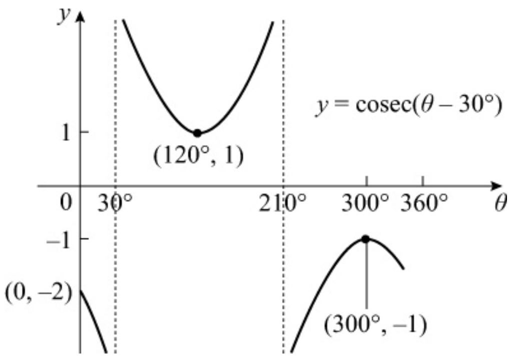


7 d Translation of $y = \operatorname{cosec}\theta$ by the vector $\begin{pmatrix} 30 \\ 0 \end{pmatrix}$

Minimum at (120°, 1)

Maximum at (300°, −1)

It meets the y-axis at $(0, -2)$

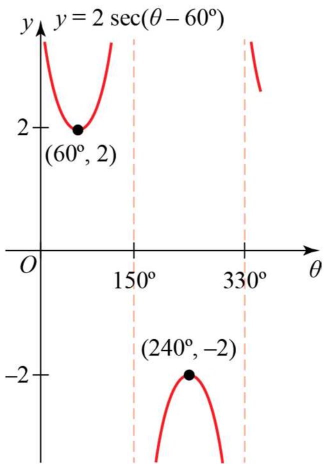


e $y = 2\sec(\theta - 60^\circ)$ is $y = \sec\theta$ translated by the vector $\begin{pmatrix} 60 \\ 0 \end{pmatrix}$ and then stretched by a scale factor 2 in the y direction.

Minimum at (60°, 2)

Maximum at $(240°, -2)$

It meets the y-axis at (0, 4)

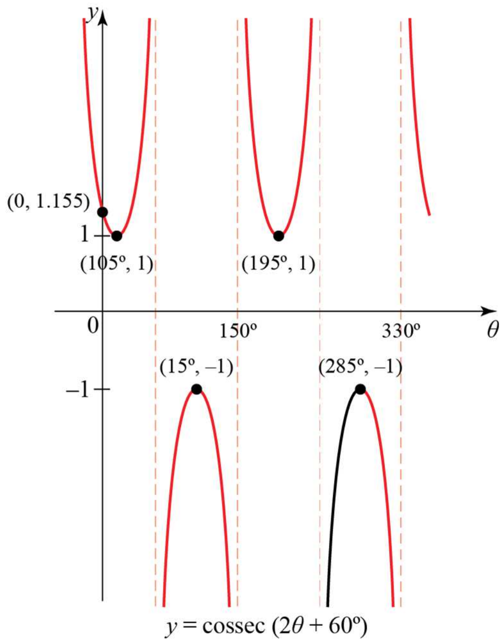


f $y = \operatorname{cosec}(2\theta + 60^\circ)$ is $y = \operatorname{cosec}\theta$ translated by the vector $\begin{pmatrix} -60 \\ 0 \end{pmatrix}$ and then stretched by a scale factor $\frac{1}{2}$ in the θ direction.

Minima at (15°, 1), (195°, 1)

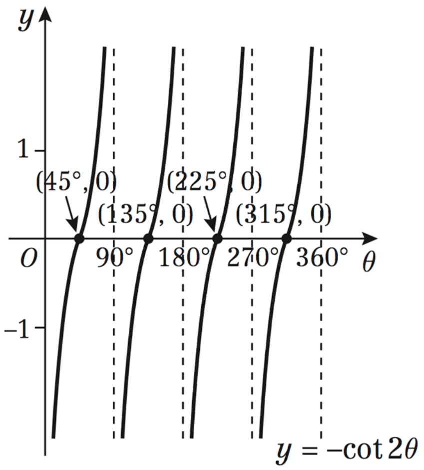
Maxima at $(105°, -1), (285°, -1)$

It meets the y-axis at $(0, 1.155)$



7 g $y = -\cot 2\theta$ is $y = \cot\theta$ stretched by a scale factor $\frac{1}{2}$ in the θ direction and then reflected in the x-axis.

It meets the θ-axis at (45°, 0), (135°, 0), (225°, 0) and (315°, 0)

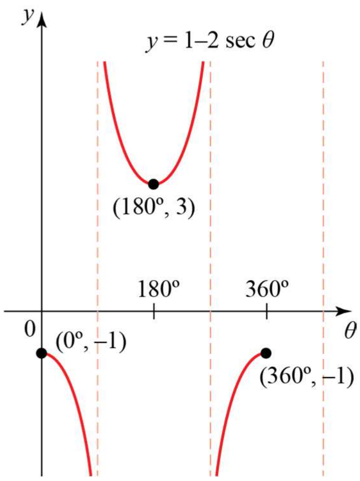


h $y = 1 - 2\sec\theta = -2\sec\theta + 1$ is $y = \sec\theta$ stretched by a scale factor 2 in the y direction, reflected in the x-axis and then translated by the vector $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$

Minima at (180°, 3)

Maxima at $(0, -1)$, $(360°, -1)$

It meets the y-axis at $(0, -1)$



8 a The period of $\sec\theta$ is 2π radians

$y = \sec 3\theta$ is a stretch of $y = \sec\theta$ with scale factor $\frac{1}{3}$ in the θ direction.

So the period of $\sec 3\theta$ is $\frac{2\pi}{3}$

b $\operatorname{cosec}\theta$ has a period of 2π

$\operatorname{cosec}\frac{1}{2}\theta$ is a stretch of $\operatorname{cosec}\theta$ in the θ direction with scale factor 2.

So the period of $\operatorname{cosec}\frac{1}{2}\theta$ is 4π

c $\cot\theta$ has a period of π

$2\cot\theta$ is a stretch in the y direction by scale factor 2. So the periodicity is not affected.

The period of $2\cot\theta$ is π

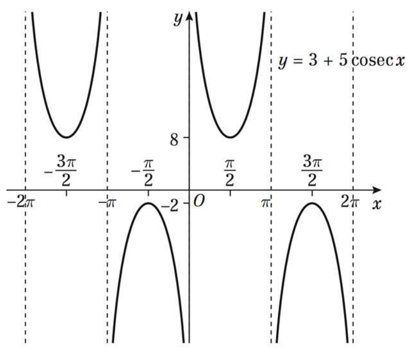
d $\sec\theta$ has a period of 2π

$\sec(-\theta)$ is a reflection in the y-axis.

So the periodicity is not affected.

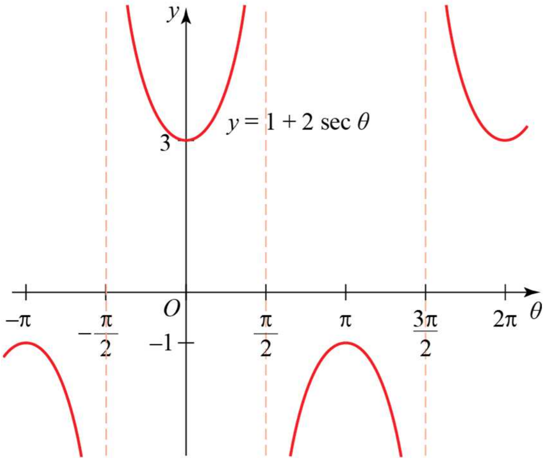
The period of $\sec(-\theta)$ is 2π

9 a $y = 3 + 5\operatorname{cosec}\theta$ is $y = \operatorname{cosec}\theta$ stretched by a scale factor 5 in the y direction and then translated by the vector $\begin{pmatrix} 0 \\ 3 \end{pmatrix}$



b $-2 < k < 8$

10 a



b The θ coordinates at points at which the gradient is zero are at the maxima and minima. These are $\theta = -\pi, 0, \pi, 2\pi$

c Minimum value of $\frac{1}{1+2\sec\theta}$

is where $1+2\sec\theta$ is a maximum.

So minimum value of $\frac{1}{1+2\sec\theta}$

is $\frac{1}{-1} = -1$

The first positive value of θ where this occurs is when $\theta = \pi$

(see diagram)

Maximum value of $\frac{1}{1+2\sec\theta}$

is where $1+2\sec\theta$ is a minimum.

So maximum value of $\frac{1}{1+2\sec\theta}$ is $\frac{1}{3}$

The first positive value of θ where this occurs is when $\theta = 2\pi$

(see diagram)